LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2024**

UMT 5503 – DISCRETE MATHEMATICS

Date: 14-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Answer the following**
a)	Using the Statements " R: Mark is rich and H: Mark is happy", write the statement "Mark is rich or unhappy" in symbolic form.
b)	Define the universe of discourse.
c)	Define Monoid homomorphism.
d)	Write any two properties of Lattices.
e)	Define Boolean algebra.
2.	**Fill in the blanks**
a)	A formula which is equivalent to a given formula and which consists of a sum of elementary products is called a ______________ of the given formula.
b)	In the quantifier $(x)\ P(x, y)$, the occurrence of y is a ______ occurrence.
c)	For a monoid $\langle M,$ ¿$\rangle$, the existence of the identity element guarantees that no two columns or rows of the composition table are ____________.
d)	The lattice $\langle L^n, \leq_n \rangle$ will be called ______________________.
e)	A boolean expression in n variables x_1, x_2, …., x_n is called symmetric if interchanging any two variables results in an ________________ expression.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Choose the best answer**
a)	Which of the following is not true. (i) $\neg\neg P$ is equivalent to P (ii) $\neg P$ is equivalent to P (iii) $P \vee P$ is equivalent to P (iv) $P \vee \neg P$ is equivalent to $Q \vee \neg Q$
b)	The implications $P, P \rightarrow Q \Rightarrow Q$ is ____________ (i) Modus Ponens (ii) Modus tollens (iii) disjunctive syllogism (iv) hypothetical syllogism
c)	An element of an alphabet is called a____________ (i) letter (ii) character (iii) symbol (iv) all the above
d)	A mapping $g\colon L \longrightarrow S$ is called a lattice homomorphism from the lattice $\langle L,$¿ $, \oplus \rangle$ to $\langle S, \wedge, \vee \rangle$if for any $a, b \in L$_________________ (i) $g(a*b)=g(a) \vee g(b)$ and $g(a \oplus b)=g(a) \vee g(b)$ (ii)$g(a*b)=g(a) \wedge g(b)$ and $g(a \oplus b)=g(a) \wedge g(b)$ (iii)$g(a*b)=g(a) \vee g(b)$ and $g(a \oplus b)=g(a) \wedge g(b)$ (iv)$g(a*b)=g(a) \wedge g(b)$ and $g(a \oplus b)=g(a) \vee g(b)$
e)	$\langle B,$¿$, \oplus \rangle$ is a lattice which satisfies $a \oplus (b*c)=(a \oplus b)*(a \oplus c)$then the lattice is called__________ (i) bounded lattice (ii) associative lattice (iii) distributive lattice (iv) complemented lattice
4.	**True or False**
a)	$(P \rightarrow Q) \rightarrow (\wedge Q)$, this is not a wff because $\wedge Q$ is not. _____________
b)	Rule US is called Universal Generalization. _______________

c)	The set of all natural numbers N is a semigroup under the operation $x * y = max\{x, y\}$.______________
d)	In a lattice $\langle L, \leq \rangle$ defined as a partially ordered set, it is possible to define two binary operations ¿ and $\oplus$ such that for any $a, b \in L$, $a * b = LUB\{a, b\}$ and $a \oplus b = GLB\{a, b\}$.________________
e)	The antiatoms are the complements of the atoms.____________
	SECTION B - K3 (CO2)
	Answer any TWO of the following in 100 words each. (2 x 10 = 20)
5.	Construct the truth table for the formula $\neg(P \vee (Q \wedge R)) \rightleftarrows ((P \vee Q) \wedge (P \vee R))$.
6.	For any commutative monoid $\langle M, ¿ \rangle$, prove that the set of idempotent elements of M forms a submonoid.
7.	Consider the set S_{30} = {1, 2, 3, 5, 6, 10, 15, 30}and the relation divides (/) be a partial ordering relation on S_{30}. (i) Draw the Hasse diagram on S_{30} with the relation divides. (ii) Determine all the upper and lower bounds of 3 and 5. (iii) Determine the GLB of 2 and 3. (iv) Determine the LUB of 5 and 6.
8.	State and Prove DeMorgan's law of Boolean algebra.
	SECTION C – K4 (CO3)
	Answer any TWO of the following in 100 words each. (2 x 10 = 20)
9.	Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.
10.	Prove that from (a) $(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$ (b) $(\exists y)(M(y) \wedge \neg W(y))$ the conclusion $(x)(F(x) \rightarrow \neg S(x))$ follows.
11.	Let $\langle L, \leq \rangle$ be a lattice defined on two binary operations of meet and join denoted by ¿ and $\oplus$ then prove that for any $a, b, c \in L$ the distributive inequality holds.
12.	Write the following boolean expressions in an equivalent sum-of-products canonical form in the three variables x_1, x_2, and x_3 : (a) x_1 ¿ x_2 (b) $x_1 \oplus x_2$ (c) $(x_1 \oplus x_2)' *$¿ x_3.
	SECTION D – K5 (CO4)
	Answer any ONE of the following in 250 words (1 x 20 = 20)
13.	(i) Show that $S \vee R$ is a tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$. (ii) Let $\langle S$, ¿$\rangle$be a given semigroup. Then prove that there exists a homomorphism $g: S \rightarrow S^s$, where $\langle S^s, \circ \rangle$is a semigroup of functions from S to S under the operation of left composition.
14.	(i) Show that every ordered Lattice satisfies the following properties of the algebraic lattice. (a) Commutative law and (b) Associative law (ii) Obtain the values of the Boolean forms (a) x_1 ¿ x_2 (b) x_1 ¿$(x_1' \oplus x_2)$ (c) $x_1 \oplus (x_1$ ¿ $x_2)$ over the ordered pairs of the two-element Boolean algebra
	SECTION E – K6 (CO5)
	Answer any ONE of the following in 250 words (1 x 20 = 20)
15.	(i) Obtain the principal disjunctive normal forms of (a) $\neg P \vee Q$ (b) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$. (ii) Show that the following premises are inconsistent. (a) If Jack misses many classes through illness, then he fails high school. (b) If Jack fails high school, then he is uneducated. (c) If Jack reads a lot of books, then he is not uneducated. (d) Jack misses many classes through illness and reads a lot of books.
16.	(i) Explain how the algebraic systems $\langle Z_m, +_m \rangle$ and $\langle Z_m, \times_m \rangle$ are monoids. (ii) Determine the lattice L_1 which is the chain of divisor 4 and lattice L_2 which is the chain of divisor 9. Hence draw the lattice $L_1 \times L_2$, where L_1 and L_2 has partial ordering relation of division.

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